Ratio Problems

Will this document help?

If the following ratio problems are easy for you to solve, probably not; otherwise, probably yes:

The Eastside Tennis Club is merging with the Westside Tennis Club to form the All City Tennis Club. The Eastside Tennis Club is three times as big as the Westside Tennis Club. If the Eastside Tennis Club has a ratio of 1 : 2 pros to novices, and the Westside Tennis Club has a ratio of 5 : 4 pros to novices, what will be the ratio of pros to novices when they merge into the All City Tennis Club?

Allie has red, blue, and green marbles in a ratio of 2 : 3 : 3. Billy has red, blue, and green marbles in a ratio of 3 : 3 : 2. Callie has red, blue, and green marbles in a ratio of 7 : 5 : 3. Allie and Billy have the same number of green marbles. Allie and Callie have the same number of blue marbles. If Allie, Billy, and Callie have 290 marbles all together, how many red marbles do each of them have?

Ratio Problems

Many people have trouble with ratio problems, so let's see if we can't find a simple way to think about ratio problems that will make at least some of them easier to solve. Let's start by considering what a ratio is.

A **ratio** is

1. the relationship between 2 or more quantities

   The meaning of a ratio is in the *relationship* between its numbers, *not* in the numbers themselves; the "*relationship* between its numbers" essentially means that no matter what the actual ratio numbers are, they still reduce to the same lowest terms.

2. written as

   (a) "the ratio of <*this*> to <*that*> is <*number*> to <*number*>" (e.g. the ratio of *girls* to *boys* is *3* to *4*),

   (b) as a fraction (e.g. $\frac{girls}{boys} = \frac{3}{4}$), or

   (c) as numbers separated by colons (e.g. *girls* : *boys* = *3* : *4*); when a ratio consists of more than 2 quantities, we usually use colon notation, e.g. *apples* : *oranges* : *grapes* = *1* : *2* : *3*

3. usually reduced to its lowest terms

   For example, if there are 30 girls and 40 boys in a classroom, the ratio of *girls* : *boys* is usually written as $\frac{3}{4}$, although $\frac{30}{40}$, $\frac{15}{20}$, and $\frac{6}{8}$ are perfectly valid too. Since nearly every ratio problem would be trivial if all of the ***unreduced*** original quantities were known, problems usually give or ask for ***reduced*** ratios; thus, we will assume all ratios are reduced to their lowest terms.

Point 3 is one of the most important, yet often under-emphasized, concepts to understand about ratios: A ratio starts out as a relationship between some ***unreduced*** original data that is then simplified by canceling out a factor common to all of the terms; I call this factor by the more generic term ***multiplier***, and not by its usual implementation, the greatest common factor, to emphasize it doesn't <u>have</u> to be the greatest common factor.[1] The ***multiplier*** is simply whatever factor is required to get back to the ***unreduced*** original data numbers. Thus, if given a ratio, to get back to the original data numbers, we need to multiply all of the ratio's terms by the ***multiplier***. To continue with our students example, if the ratio of *girls* : *boys* in a classroom is $\frac{3}{4}$ and we know the ***multiplier*** is 10, we can calculate that there are $3 \cdot 10 = 30$ girls and $4 \cdot 10 = 40$ boys in the classroom. ***The multiplier allows us to convert unreduced original data terms to reduced ratio terms and vice versa.***

Notice that simplifying a ratio is just the normal method to reduce a fraction; e.g. given our previous example of $\frac{girls}{boys} = \frac{30}{40}$ in a classroom, the greatest common factor of 30 and 40 is 10, so this is the ***multiplier*** that is canceled out of both terms to reduce the ratio to its lowest level, $\frac{3}{4}$. If nothing can be canceled out (i.e., at least two of the numbers are relatively prime and therefore their greatest common factor is 1), the ***multiplier*** is 1 and the ***unreduced*** quantities are equal to the ***reduced*** quantities.

[1] The ***multiplier*** is dependent on how the ***unreduced*** original ratio was simplified. Given original data of 30 girls and 40 boys, there are many ways to write a *girls* : *boys* ratio and each way has its own ***multiplier***; for example,

Ratio	*Multiplier*
30 : 40	1
15 : 20	2
6 : 8	5
3 : 4	10

Two common constructions of ratios are

1. ***part* : *part***

   Examples: a *girls* : *boys* ratio of students in a classroom is a ratio of ***parts*** that make up the ***whole*** of *students in a classroom* and a *red* : *blue* : *green* : *yellow* ratio of marbles in a bag is a ratio of ***parts*** that make up the whole of marbles in a bag

2. ***part* : *whole***

   Examples: a *girls* : *students in a classroom* ratio is a ***part*** : ***whole*** ratio since *girls* are one ***part*** of *students in a classroom* and a ratio of *red marbles : total number of marbles in a bag* is a ***part*** : ***whole*** ratio since *red marbles* are just one ***part*** of *a bag of marbles*

Given enough information, we can convert between ***part* : *part*** and ***part* : *whole*** representations. If we are given ***part* : *part*** information for all of the ***parts*** that make up the ***whole***, we can calculate the ***whole*** by simply adding up all of the ***parts***. For example, if we are told the ***reduced part* : *part*** ratio of *girls* to *boys* in a classroom is *3* : *4*, we know the ***reduced*** ratio information for all of the ***parts*** (since all the students are either girls or boys) and therefore can add the ***reduced parts*** to calculate the ***reduced whole*** (3 + 4 = 7 students); thus, the ***reduced part* : *whole*** ratios are *girls* : *students = 3* : *7* and *boys* : *students = 4* : *7*. Conversely, if we are told the ***reduced part* : *whole*** ratio of *boys* to *students* is *4* : *7*, we can subtract the ***part*** from the ***whole*** to get the other ***part*** (7 – 4 = 3 girls); therefore, the other ***reduced part* : *whole*** ratio is *girls* : *students = 3* : *7* and the ***reduced part* : *part*** ratio of *girls* : *boys* is *3* : *4*.

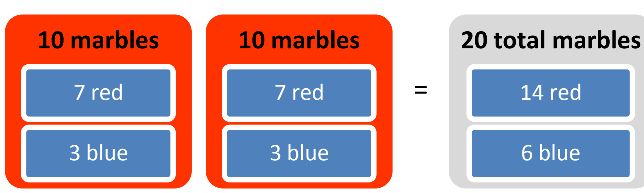
When working ratio problems, it is important to be clear on if a number is (1) an ***unreduced*** or a ***reduced*** quantity and (2) associated with a ***part*** or a ***whole***.

Finally, we can think of ratios as groupings. Suppose a ratio problem states

If there are only 14 red marbles and 6 blue marbles in a bag, what is the ratio of red to blue marbles?

The ratio of *red* to *blue* marbles is easily calculated as $\frac{red}{blue} = \frac{14}{6} = \boxed{\frac{7}{3}}$. We can think of the ratio as a group: For every 7 red marbles, there are 3 blue marbles (or for every 10 marbles, 7 are red and 3 are blue):

The ***multiplier***, which in this case is 2, tells us how many of these groups of 7 red marbles and 3 blue marbles that we have:



Note that the ratio of $\frac{7}{3}$ *red* to *blue* marbles doesn't tell us the actual number of red or blue marbles in the bag; we need the ***multiplier*** to get back to the original quantities: $\frac{red}{blue} = \frac{7\ x\ 2}{3\ x\ 2} = \frac{14}{6}$.

Suppose the problem then states

> If 3 red marbles are removed from the bag and 2 blue marbles are added, what is the new ratio of red to blue marbles?

It is ***very*** important to understand that we can add or subtract quantities from the ***unreduced*** original data, but that we can't do so from the ***reduced*** ratio's terms. We calculate the new ratio by adding and subtracting the changes from the ***unreduced*** original quantities: $\frac{red}{blue} = \frac{14-3}{6+2} = \boxed{\frac{11}{8}}$. We can never add or subtract ***unreduced*** original quantity changes from a ***reduced*** ratio's terms; this is wrong: $\frac{red}{blue} = \frac{7-3}{3+2} = \frac{4}{5}$. Always remember: ***Never add or subtract from a reduced ratio's term; only add or subtract from the unreduced original data.***

Warning: The order the ratio is written in is important. If a problem states the ratio of *girls* to *boys* is *3* : *4*, the *3* has to be associated with *girls* and the *4* has to be associated with *boys*; it is correct to write $\frac{girls}{boys} = \frac{3}{4}$ or $\frac{boys}{girls} = \frac{4}{3}$, but incorrect to write $\frac{girls}{boys} = \frac{4}{3}$ or $\frac{boys}{girls} = \frac{3}{4}$.

Basic Ratio Problems

A basic ratio problem will usually (directly or indirectly) state 2 out of the 3 components of the equation

original quantity = ratio quantity x ***multiplier***

for either a ***part*** or the ***whole*** and we have to calculate the 3rd component. Often the first calculations are to find the ***multiplier***, which we then use to calculate the desired quantity or ratio. Once you know the ***multiplier*** for a ***whole*** or ***part*** quantity, you know the ***multiplier*** for all of the related ***parts*** and the ***whole***. This is why the mantra for basic ratio problems is ***Know the multiplier, know everything***.

Let's look at a basic ratio problem:

> *In a class of 45 students, the ratio of boys to girls is 5 : 4. How many girls are in the class?*

If you're already familiar with basic ratio problems, you're going "$5+4=9,\ 45 \div 9 = 5,\ 4 \times 5 = \boxed{20}$ girls"—and that's correct. Here's a more detailed solution:

Solution
We know *original quantity of girls = ratio quantity of girls x* ***multiplier*** (1) **what we want to calculate** **given as 4** **unknown** We need to determine the ***multiplier***. To do so, we need to use the other information the problem gives us: *original quantity of students = ratio quantity of students x* ***multiplier*** (2) **given as 45** **we can deduce this!** **unknown** The ratio quantity of *students* (the ***whole***) is the sum of the ratio quantities of *boys* and *girls* (its ***parts***): 5 + 4 = 9 students. We plug this into equation (2) and calculate the ***multiplier***: 45 = 9 x ***multiplier***; ***multiplier*** = 5. Plugging the ***multiplier*** into equation (1), we calculate the original quantity of girls = 4 *x* 5 = $\boxed{20}$ girls.

Notice that if we wanted to also calculate the number of boys in the problem above, we'd use the same ***multiplier***, 5, to get $5 \times 5 = \boxed{25}$ boys.

At a minimum, we should use the ratio information to sanity-check our answer. Since the problem states "*the ratio of boys to girls is 5 : 4*," the total number of boys *must* be a multiple of 5, the total number of girls *must* be a multiple of 4, and the total number of students *must* be a multiple of 9 (= 5 + 4).

Let's look at one more basic ratio problem:

The ratio of 2 numbers is 7 : 4. The difference between the 2 numbers is 18. What is the smaller number?

Solution
Let *A* and *B* represent the 2 numbers. Let's represent the original data of the 2 numbers as $\frac{A}{B} = \frac{7m}{4m}$, where m = ***multiplier***. This means that A was originally $7m$ and B was originally $4m$. The problem states that $A - B = 18$. Therefore, $7m - 4m = 3m = 18; m = 6$. The smaller number is $4m = 4 \cdot 6 = \boxed{24}$.

If you have a good understanding of ratios, you can solve most basic ratio problems in your head.

More Complex Ratio Problems

Since even the most complex ratio problems would be trivial if we knew all of the ***unreduced*** original quantities, when we are given ***reduced*** ratio quantities, it is often helpful to use the ***multiplier*** to convert ratio quantities to the original quantities. The ***multiplier*** may never be directly calculated, but its presence often makes the problem easier to solve.

One type of ratio problem deals with *before* and *after* sets of circumstances; usually some information is given about the initial quantities of some things, then those quantities are changed, and then some information is given about the resultant quantities. Here's an example of this type of problem:

At the beginning of a yard sale, the ratio of toys to books was 3 : 5. During the yard sale, 50% of the toys and 60% of the books were sold, which left a total of 70 toys and books unsold. How many toys were there before the yard sale?

Solution A: 2 equations, 2 unknowns	**Solution B: Using the *Multiplier***
We can write 2 independent equations and 2 unknowns, so we can solve this system of equations. There are a couple of ways to solve this; we'll do it by solving simultaneous equations: Let *T* = the number of toys *before* the yard sale and *B* = the number of books *before* the yard sale. $\frac{T}{B} = \frac{3}{5}$ $5T = 3B$ $-5T + 3B = 0. \ (1)$ The problem states that $unsold\ toys + unsold\ books = 70$ $\frac{50}{100}T + \frac{40}{100}B = 70$ $\frac{1}{2}T + \frac{2}{5}B = 70$ Multiplying the whole equation by 10 gives $5T + 4B = 700. \quad (2)$ Solving equations (1) and (2), we get $-5T + 3B = 0$ $\underline{5T + 4B = 700}$ $7B = 700$ $B = 100$ Using $5T = 3B$ and plugging in for *B*, we get $5T = 3(100)$ $T = \frac{3(100)}{5}$ $T = 3 \cdot 20 \quad (3)$ $T = \boxed{60}$	We represent the original data *before* the yard sale as $\frac{toys}{books} = \frac{3m}{5m}$ where m = ***multiplier*** for the *before* quantities. $unsold\ toys = \left(\frac{50}{100}\right)(3m) = \frac{3}{2}m.$ $unsold\ books = \left(\frac{40}{100}\right)(5m) = 2m.$ The problem states that $unsold\ toys + unsold\ books = 70$ $\frac{3}{2}m + 2m = 70$ $\frac{3+4}{2}m = 70$ $\frac{7}{2}m = 70$ $m = 70 \cdot \frac{2}{7}$ $m = 20$ Hence, the number of toys *before* the yard sale is $3m = 3 \cdot 20 = \boxed{60}$. In this method, we directly calculate the *before* ***multiplier***, which results in a simple and short solution. ***Know the multiplier, know everything.***

Notice in Solution A that equation (3) is the *original quantity of toys = ratio quantity of toys x* ***multiplier*** and that this method also calculates the *before* ***multiplier*** as 20; unless we're explicitly dealing with the ***multiplier***, we don't normally notice these details.

Another typical ratio problem that deals with *before* and *after* quantities is

> *Bob has some blue and green marbles. The ratio of blue to green marbles is 5 : 3. After Bob trades 2 blue marbles for 3 green marbles, the ratio of blue to green marbles is 6 : 5. How many marbles did Bob have originally?*

Using the information given in the problem, we write the following equations: $\frac{B}{G} = \frac{5}{3}$ (*before*) and $\frac{B-2}{G+3} = \frac{6}{5}$ (*after*), where *B* = Bob's original quantity of blue marbles and *G* = Bob's original quantity of green marbles.

<table>
<tr><th>Solution A: 2 equations, 2 unknowns</th><th>Solution B: Using the Multiplier</th></tr>
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We have 2 independent equations and 2 unknowns, so we can solve this system of equations. There are a couple of ways to solve this; we'll do it using substitution:

$\frac{B}{G} = \frac{5}{3} \quad (before)$

$B = \frac{5}{3}G$

$\frac{B-2}{G+3} = \frac{6}{5} \quad (after)$

Cross multiplying gives

$5(B-2) = 6(G+3)$

$5B - 10 = 6G + 18$

$5B = 6G + 28$

Plugging in $\frac{5}{3}G$ for B gives

$5\left(\frac{5}{3}G\right) = 6G + 28$

$\frac{25}{3}G = 6G + 28$

$\frac{25-18}{3}G = 28$

$\frac{7}{3}G = 28$

$G = \frac{3}{7} x\, 28$

$G = 3\, x\, 4 = 12 \quad (1)$

Plugging in 12 for G gives

$B = \frac{5}{3} x\, 12$

$B = 5\, x\, 4 = 20 \quad (2)$

Therefore, Bob originally had 20 blue marbles and 12 green marbles, for a total of $\boxed{32}$ marbles, which is our answer.

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$$\frac{B-2}{G+3} = \frac{6}{5}$$

We note that since $\frac{B}{G} = \frac{5}{3}$ was derived from $\frac{B}{G} = \frac{5m}{3m}$ (the ***unreduced*** original data),

$$B = 5m$$
$$G = 3m$$

where m = ***multiplier*** for the *before* quantities.

Plugging these in gives

$$\frac{5m-2}{3m+3} = \frac{6}{5}$$

We now have 1 equation with 1 unknown; cross multiplying gives

$$5(5m-2) = 6(3m+3)$$
$$25m - 10 = 18m + 18$$
$$7m = 28$$
$$m = 4$$

Therefore, $B = 5\, x\, 4 = 20$ and $G = 3\, x\, 4 = 12$. The total number of marbles is $20 + 12 = \boxed{32}$.

In this method, we directly calculate the *before* ***multiplier***. This solution is much simpler, the math is less complicated, and thus the solution is overall less prone to error.

Know the multiplier, know everything.

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Notice in Solution A that equations (1) and (2) are the *original quantity = ratio quantity x* ***multiplier*** and that the calculations for *G* and *B* both determine that the *before* ***multiplier*** is 4. The *before* ***multiplier*** is again so buried in the calculations that it's very easy to miss.

Since the *before* and *after* ratios are unrelated (because they are derived from different quantities), they have different ***multipliers***. Let's calculate the *after* ***multiplier***: Using the *after* information for the blue marbles, we calculate the *original quantity = ratio quantity x* ***multiplier*** as 18 = 6 *x* ***multiplier***; therefore, the *after* ***multiplier*** is 3.

Another type of ratio problem is one in which two or more ratios are combined into a resultant ratio. For example,

> *The Eastside Tennis Club is merging with the Westside Tennis Club to form the All City Tennis Club. The Eastside Tennis Club is three times as big as the Westside Tennis Club. If the Eastside Tennis Club has a ratio of 1 : 2 pros to novices, and the Westside Tennis Club has a ratio of 5 : 4 pros to novices, what will be the ratio of pros to novices when they merge into the All City Tennis Club?*

For both solutions below, let

E = the total number of players in the Eastside Tennis Club and

W = the total number of players in the Westside Tennis Club.

We also note that *E* = 3*W* since the problem states that the Eastside Tennis Club is 3 times as big as the Westside Tennis Club.

Both strategies below find ways to represent the original Eastside and Westside data and then add those to form the All City original data (remember: we can only add original data quantities); the resultant All City *pro* : *novice* expression is then simplified to get the answer.

Solution A: Not Using *Multipliers*

Since *E* = 3*W*, $W = \frac{E}{3}$.

The total number of players in the Eastside Tennis Club is a multiple of 3 (= 1 + 2, i.e. the ***whole*** = the sum of its ***parts***). The ***part*** : ***whole*** ratios then say that $\frac{1}{3}$ of the Eastside total is pros and $\frac{2}{3}$ of the Eastside total are novices.

Therefore, the ***unreduced*** original data for the Eastside Tennis Club is

$$Eastside\ pros = \frac{E}{3} \quad \text{and} \quad Eastside\ novices = \frac{2E}{3}$$

The total number of players in the Westside Tennis Club is a multiple of 9 (= 5 + 4, i.e. the ***whole*** = the sum of its ***parts***). The ***part*** : ***whole*** ratios then say that $\frac{5}{9}$ of the Westside total are pros and $\frac{4}{9}$ of the Westside total are novices.

Therefore, the ***unreduced*** original data for the Westside Tennis Club is

$$Westside\ pros = \frac{5}{9}W = \frac{5}{9}\cdot\frac{E}{3} = \frac{5E}{27}$$

and

$$Westside\ novices = \frac{4}{9}W = \frac{4}{9}\cdot\frac{E}{3} = \frac{4E}{27}$$

When the 2 clubs merge, the ***unreduced*** result will be

$$All\ City\ pros = \frac{E}{3} + \frac{5E}{27} = \frac{9E}{27} + \frac{5E}{27} = \frac{14E}{27}$$

Solution B: Using *Multipliers*

Let *m* be the ***multiplier*** for the Eastside Tennis Club ratio.

The ***unreduced*** original data for the Eastside Tennis Club is

$$\frac{Eastside\ pros}{Eastside\ novices} = \frac{1m}{2m}$$

The total number of players in the Eastside Tennis Club is therefore *E* = 1*m* + 2*m* = 3*m*.

Let *n* be the ***multiplier*** for the Westside Tennis Club.

The ***unreduced*** original data for the Westside Tennis Club is

$$\frac{Westside\ pros}{Westside\ novices} = \frac{5n}{4n}$$

The total number of players in the Westside Tennis Club is therefore *W* = 5*n* + 4*n* = 9*n*.

Since E = 3W, 3*m* = 3(9*n*) = 27*n*. Solving for *m*, $m = 9n$.

When the 2 clubs merge, the ***unreduced*** result will be

$$\frac{All\ City\ pros}{All\ City\ novices} = \frac{1m+5n}{2m+4n} = \frac{1(9n)+5n}{2(9n)+4n} = \frac{9n+5n}{18n+4n} = \frac{14n}{22n} = \boxed{\frac{7}{11}},$$

which is our answer.

Our mantra for basic ratio problems is ***Know the multiplier, know everything***, but in this more complex ratio problem, we can't. We weren't given enough information to calculate either of the ***multipliers*** *m* or *n*.

and $$All\ City\ novices = \frac{2E}{3} + \frac{4E}{27} = \frac{18E}{27} + \frac{4E}{27} = \frac{22E}{27}.$$ We now write these as a ratio: $$\frac{All\ City\ pros}{All\ City\ novices} = \frac{\left(\frac{14E}{27}\right)}{\left(\frac{22E}{27}\right)} = \frac{14E}{27} \div \frac{22E}{27} = \frac{14E}{27} \cdot \frac{27}{22E} = \frac{14}{22} = \boxed{\frac{7}{11}},$$ which is our answer.	Even so, we used the fact that there <u>*are*</u> ***multipliers*** to easily calculate the answer.

Once again, using ***multipliers*** to represent the original data made the problem easier to solve and required less complicated math.

So this is an easier way to think about ***some*** ratio problems, especially those that involve only basic ratio concepts or those that involve changes to original quantities:

Convert all ratios to the original data by reintroducing the multipliers.

Sometimes we will be able to explicitly calculate the original data; other times we will only be able to represent the original data as a product of a ratio quantity times a ***multiplier***. Either way, having some form of the original data often makes these types of problems easier to solve—and that's our goal.

Practice Problems

Solutions are on the next pages.

1. *A carton of a dozen eggs was dropped and now the ratio of cracked to uncracked eggs is 1 : 2. How many eggs are uncracked?*

2. *The ratio of 2 numbers is 3 : 11. The sum of the 2 numbers is 238. What is the smaller number?*

3. *I am going to the bank to exchange some nickels for quarters. Before I go to the bank, the ratio of nickels to quarters is 7 : 3. After I exchange 45 nickels for quarters, the ratio of quarters to nickels will be 3 : 1. How much money do I have?*

4. *Archie and Bob each have a bag of marbles with only green and blue marbles in it. Archie's ratio of green to blue marbles is 3 : 2. Bob's ratio of green to blue marbles is 5 : 7. Bob has four-fifths as many marbles as Archie. What is the ratio of Archie's quantity of green marbles to Bob's quantity of green marbles?*

5. *Allie has red, blue, and green marbles in a ratio of 2 : 3 : 3. Billy has red, blue, and green marbles in a ratio of 3 : 3 : 2. Callie has red, blue, and green marbles in a ratio of 7 : 5 : 3. Allie and Billy have the same number of green marbles. Allie and Callie have the same number of blue marbles. If Allie, Billy, and Callie have 290 marbles all together, how many red marbles do each of them have?*

6. *A 12-hour clock with an AM/PM indicator is set to the correct time at 3:00 PM. Unfortunately, the clock runs fast and gains 4 minutes every hour. What time will it really be when this clock next reads 3:00 PM?*

Solutions

1. $\frac{cracked\ eggs}{uncracked\ eggs} = \frac{1m}{2m}$, where m = ***multiplier***.

   The total number of eggs is 12 = $1m + 2m = 3m$; $m = 4$. Therefore, there are $2m = 2 \cdot 4 = \boxed{8}$ uncracked eggs.

2. Let *A* and *B* represent the 2 numbers.
   Let's represent the original data of the 2 numbers as $\frac{A}{B} = \frac{3m}{11m}$, where m = ***multiplier***.

   This means that $A = 3m$ and $B = 11m$.

   The problem states that $A + B = 238$. Therefore, $3m + 11m = 14m = 238; m = 17$.

   The smaller number is $3m = 3 \cdot 17 = \boxed{51}$.

3. Did you catch that the *after* ratio information is flipped in the problem? The *before* ratio is given as *nickels* : ***quarters***, but the *after* ratio is given as ***quarters*** : *nickels*—check that ratio terms are associated with what they represent, regardless of how the problem states the information.

   If you didn't notice that the information wasn't given in the same order, you should have gotten a negative ***multiplier*** if you didn't flip the *after* information (-36, a definite red flag since a negative ***multiplier*** doesn't make sense for this problem[2]) or $136.80 if you flipped the *before* information.

   If you didn't catch this, try the problem again.

   The rest of the solution is on the next page.

[2] A ratio might have negative terms in situations where some quantity owed, missing, below a baseline, or some such circumstance is defined to be negative. When working ratio problems, unless a problem very clearly states negative quantities, you should be concerned and recheck your work if you get a negative ***multiplier***.

3. (con't) Since 5 nickels = 1 quarter, we know that 45 nickels will be exchanged for 9 quarters.

The *before* ratio is $\frac{nickels}{quarters} = \frac{7m}{3m}$, where m = *before* ***multiplier***.

The *after* ratio is $\frac{nickels}{quarters} = \frac{1}{3} = \frac{7m-45}{3m+9}$.

Cross multiplying gives

$$3(7m - 45) = 3m + 9$$
$$21m - 135 = 3m + 9$$
$$18m = 144$$
$$m = 8$$

Therefore, I have $7 \cdot 8 = 56$ nickels and $3 \cdot 8 = 24$ quarters.

Calculating my total amount of money *T*:

$$T = (56 \cdot \$0.05) + (24 \cdot \$0.25)$$
$$T = \$2.80 + \$6.00$$
$$T = \boxed{\$8.80}$$

4. Let m = Archie's ***multiplier*** and n = Bob's ***multiplier***:

$$\frac{Archie_{green}}{Archie_{blue}} = \frac{3m}{2m} \qquad \frac{Bob_{green}}{Bob_{blue}} = \frac{5n}{7n} \qquad Bob_{total} = \frac{4}{5} \cdot Archie_{total}, \therefore \frac{Bob_{total}}{Archie_{total}} = \frac{4}{5}$$

$$\frac{Bob_{total}}{Archie_{total}} = \frac{5n + 7n}{3m + 2m} = \frac{12n}{5m} = \frac{4}{5}$$

$$60n = 20m$$
$$3n = m$$
$$\frac{Archie_{green}}{Bob_{green}} = \frac{3m}{5n} = \frac{3(3n)}{5n} = \boxed{\frac{9}{5}}$$

Extra: In the comments on *mathnotations.blogspot.com/2008/02/beyond-mixed-nuts-more-challenging.html*, Dave Marain uses a very nice method to solve certain ***part*** : ***whole*** ratio problems:

From $\frac{Archie_{green}}{Archie_{blue}} = \frac{3}{2}$, we conclude $\frac{Archie_{green}}{Archie_{total}} = \frac{3}{5}$.

From $\frac{Bob_{green}}{Bob_{blue}} = \frac{5}{7}$, we conclude $\frac{Bob_{green}}{Bob_{total}} = \frac{5}{12}$.

Using these 2 new ratios with $\frac{Bob_{total}}{Archie_{total}} = \frac{4}{5}$,

$$\frac{Archie_{green}}{Bob_{green}} = \left(\frac{Archie_{green}}{Archie_{total}}\right)\left(\frac{Archie_{total}}{Bob_{total}}\right)\left(\frac{Bob_{total}}{Bob_{green}}\right) = \left(\frac{3}{5}\right)\left(\frac{5}{4}\right)\left(\frac{12}{5}\right) = \boxed{\frac{9}{5}}$$

5. Let's use a table to organize the information and let a = Allie's ***multiplier***, b = Billy's ***multiplier***, and c = Callie's ***multiplier***:

	red marbles	**blue marbles**	**green marbles**	**total marbles**
Allie	$2a$	$3a$	$3a$	$8a$
Billy	$3b$	$3b$	$2b$	$8b$
Callie	$7c$	$5c$	$3c$	$15c$
				290

The green cells indicate that Allie and Billy have the same number of green marbles ($3a = 2b$) and the blue cells indicate that Allie and Callie have the same number of blue marbles ($3a = 5c$).

Let's use the information the problem gives us to write b and c in terms of a*:*

$$3a = 2b; b = \frac{3}{2}a$$

$$3a = 5c; c = \frac{3}{5}a.$$

$$8a + 8b + 15c = 290$$
$$8a + 8\left(\frac{3}{2}a\right) + 15\left(\frac{3}{5}a\right) = 290$$

$$8a + 12a + 9a = 290$$
$$29a = 290$$
$$a = 10$$

Solving for the other ***multipliers***,

$$b = \frac{3}{2}a = \frac{3}{2}\cdot 10 = 15$$

$$c = \frac{3}{5}a = \frac{3}{5}\cdot 10 = 6$$

Therefore,

$$\textit{Allie has } 2a = 2 \cdot 10 = \boxed{20} \textit{ red marbles}$$
$$\textit{Billy has } 3b = 3 \cdot 15 = \boxed{45} \textit{ red marbles}$$
$$\textit{Callie has } 7c = 7 \cdot 6 = \boxed{42} \textit{ red marbles}.$$

6. This is a bit of a trick question, since the solution to this problem doesn't use ***multipliers*** (in the sense used in this document). I include it here to keep things in perspective: Not *all* types of ratio problems (here, a proportion type) can be solved using ***multipliers***. As with any tool, ***multipliers*** are very useful in the right situations and, naturally, not very useful in the wrong situations.

$$\frac{correct\ clock}{fast\ clock}: \frac{60\ min}{64\ min} = \frac{x}{24\ hr}$$

$$64x = (60)(24)\ hr$$

$$x = \frac{(60)(24)}{64}\ hr$$

$$x = \frac{45}{2}\ hr = 22\frac{1}{2}\ hr$$

Thus, only $22\frac{1}{2}$ hours will have really passed, so the correct time will be $\boxed{1{:}30\ PM}$.

Happy solving!